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ASYMPTOTIC DECOMPOSITION METHOD FOR SOLVING MILDLY NONLINEAR DIFFERENTIAL EQUATION

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In this paper Adomian's decomposition scheme is presented as a good alternative method for solving two dimensional nonlinear partial differential equation on general domain.
Comparing the scheme with the existing results obtained from finite element p and h version, shows that the present approach is good and converges rapidly.

Keywords: Adomian decomposition; Mutually non linear differential equation

C. R. Category: G1.8

1. INTRODUCTION

In this section we first describe the main algorithm of Adomian's decomposition as it is applies to a general non linear equation of the form

$$u = N(u) + f(x, y) \tag{1.1}$$

where N is a non linear operator on Hilbert space H, and f is an element of H, we first introduce a modified version of the well known Adomion's decomposition algorithm which has been recently employed to solve a wide range of problems involving algebraic, differential, integral equations.

We adapt the algorithm for solving the most general form of the two dimensional problem of the form (1.1).

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where the Adomian polynomials in (1.7) are modified to handle nonlinearities of the form e^u.

The solutions obtained is generated by using Mathematica.

2. ILLUSTRATION OF THE METHOD

In this section we shall consider an example where in (1.1) g is assumed to be e^u. The outcome of Adomian's decomposition method shall be compared with the solution obtained by using the p and h versions of F.E.M. Consider the mildly nonlinear elliptic partial differential equation of the form:

$$\frac{\partial^2}{\partial x^2}(u) + \frac{\partial^2}{\partial y^2}(u) = e^u + f(x, y) \tag{1.8}$$

where the exact solution $u(x, y) = \operatorname{Sin} \alpha \times \operatorname{Sin} \beta y$ which is adapted from a real world problem.

Following the Adomian's decomposition analysis (see [2, 3]), we define the operators

$$Lx = \frac{\partial^2}{\partial x^2}, \quad Ly = \frac{\partial^2}{\partial y^2}$$

with $N(u) = e^u$, (1.8) can be written in terms of the operator

$$e^u = L_x[u] + L_y[u] - f(x, y)$$

with $g(u) = e^u$, we get

$$\begin{aligned} A_0 &= e^{u_0} \\ A_1 &= u_1 e^{u_0} \\ A_2 &= \left[u_2 + \frac{1}{2}u_1^2\right] e^{u_0} \\ A_3 &= \left[u_3 + u_1 u_2 + \frac{u_3^3}{3!}\right] e^{u_0} \\ &\vdots \quad \vdots \end{aligned} \tag{1.9}$$

from (1.2) and (1.3), this gives

$$A_0 + A_1 + A_2 + \cdots = \sum_{i=0}^{\infty} Lx[u_i] + \sum_{i=0}^{\infty} Ly[u_i] - f(x, y) \tag{1.10}$$

TABLE III Error obtained using decomposition method on the indicated interval using four iteration

x	0.05	0.10	.15	.20	.25
.1	1.000×10^{-5}	1.1121×10^{-5}	1.2131×10^{-5}	1.4761×10^{-4}	2.451×10^{-4}
.2	2.1341×10^{-5}	2.3112×10^{-5}	6.1211×10^{-5}	52311×10^{-4}	4.8761×10^{-4}
.3	2.2113×10^{-5}	2.4661×10^{-5}	6.2112×10^{-4}	5.7614×10^{-3}	6.7134×10^{-3}
.4	2.6276×10^{-4}	3.4791×10^{-4}	6.7141×10^{-4}	5.9841×10^{-3}	6.8861×10^{-3}
.5	2.911×10^{-3}	4.7761×10^{-3}	5.1212×10^{-3}	6.2891×10^{-3}	8.2211×10^{-3}

polynomial is increased (p version) and the error obtained using the new decomposition method.

Table III shows the error obtained by using the approximation ϕ_3 and ϕ_4 as defined in (1.12), respectively. It is evident that the error decreases as the number of iterations increase, the convergence is good and the errors are small.

The results confirm that the decomposition method is of high standard and quite accurate within a few terms of (1.12).

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